

## Reminders

1. Take home open book quiz

- No help from other students

- Due April 1, 10 AM (in class or email to grader)

2. Midterm Exam

- Solutions posted on Web

- Manya is/has been out of town

Hypothesis tests and confidence intervals for  $\pi$  (population proportion)

Mean of  $\pi$  estimated by  $\hat{p} = \frac{d}{n}$

Variance of  $\pi$  is  $\frac{\pi(1-\pi)}{n}$ , estimated by  $\frac{\hat{p}(1-\hat{p})}{n}$  or  $\frac{p_0(1-p_0)}{n}$

- Comes from binomial distribution

Ex 1: 25 rats were sleep deprived for 48 hours, at which time 16 had decreased body temperature.

1) Calculate an approximate 95% CI for  $\pi$

Normal approximation to the Binomial

- Need  $n\pi > 5$ ,  $n(1-\pi) > 5$  (Rule of Thumb)

- $\pi$  is approx. Normal with mean estimate  $\frac{d}{n}$   
Var estimate  $\frac{\hat{p}(1-\hat{p})}{n}$

$$\hat{p} = \frac{16}{25} = 0.6$$

$$\frac{\hat{p}(1-\hat{p})}{n} = .0096$$

In general, the CI is given by

estimate  $\pm Z_{1-\alpha/2} \times \text{std error}$

Here, we have ( $\alpha=.05$ )

$$0.6 \pm 1.96 \times \sqrt{.0096}$$

$$\hat{p} \pm Z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= [.41, .79]$$

We are 95% confident that 41% to 79% of sleep deprived rats will have lower body temperature after 48 hours.

$$(2) \text{ Test } H_0: \pi = 0.5$$

$$H_A: \pi \neq 0.5$$

Again, use the Normal approximation

$$n\pi > 5, \text{ and } n(1-\pi) > 5$$

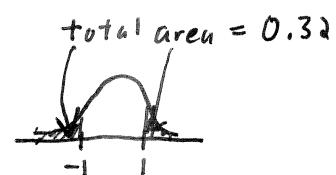
$$Z = \frac{\text{estimate} - \text{hypothesized value}}{\text{std. error of numerator}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$= \frac{0.6 - 0.5}{\sqrt{\frac{.5(1-.5)}{25}}} = \frac{.1}{.1}$$

$$= 1$$



$$p\text{-value} = 0.32$$



Note: Different std. error estimates

- Confidence interval uses  $\hat{p} = 0.6$
- Hypothesis test uses  $p_0 = 0.5$ 
  - $Z$  calculated assuming null hypothesis is true