3

$\mu_x = E(X)$ $\quad \sigma_x^2 = Var(X)$ $\quad \rho = Corr(X,Y) = \dfrac{cov(X,Y)}{\sigma_x \sigma_y}$

$\mu_y = E(Y)$ $\quad \sigma_y^2 = Var(Y)$

a

$$
\begin{aligned}
Cov(X,Y) &= E[(X-\mu_x)Y] \\
&= E[XY - \mu_x Y] \\
&= E(XY) - \mu_x E(Y) \\
&= E(XY) - E(X)E(Y) \\
&= Cov(X,Y) \quad \blacksquare
\end{aligned}
$$

b

$$E(Y|X) = aX + b$$

$$\mu_Y = E(Y) = E(E(Y|X)) = E_x(aX+b) = \boxed{a\mu_x + b}$$

c

$$E_x\{E_{Y|X}[(X-\mu_x)Y|X]\} = a\sigma_x^2$$

$$
\begin{aligned}
&= E_x\{(X-\mu_x)E_{Y|X}[Y|X]\} = E_x\{(X-\mu_x)(aX+b)\} \\
&= E_x\{aX^2 + Xb - a\mu_x X - \mu_x b\} \\
&= aE(X^2) + bE(X) - a\mu_x E(X) - \mu_x b \\
&= a(\sigma_x^2 + \mu_x^2) + \cancel{b\mu_x} - a\mu_x^2 - \cancel{b\mu_x} \\
&= a\sigma_x^2 + \cancel{a\mu_x^2} - \cancel{a\mu_x^2} \\
&= \boxed{a\sigma_x^2} \quad \blacksquare
\end{aligned}
$$