$\boxed{1}$ $P(D_1 | W) = P_1$

$P(D_2 | \bar{D}_1, W) = P_2$

where D_1 = death during surgery

D_2 = death in 2 months after surgery

W = woman

$\boxed{a}$ $P(\bar{D}_2, \bar{D}_1 | W) = P(\bar{D}_2 | \bar{D}_1, W) P(\bar{D}_1 | W)$

$= (1 - P(D_2 | \bar{D}_1, W))(1 - P(D_1 | W))$

$= \boxed{(1 - P_2)(1 - P_1)}$

$\boxed{b}$ $P(D_1 | \bar{W}) = P_1$

$P(D_2 | \bar{D}_1, \bar{W}) = 2P_2$

$P(\bar{D}_2, \bar{D}_1 | \bar{W}) = P(\bar{D}_2 | \bar{D}_1, \bar{W}) P(\bar{D}_1 | \bar{W})$

$= (1 - P(D_2 | \bar{D}_1, \bar{W}))(1 - P(D_1 | \bar{W}))$

$= \boxed{(1 - 2P_2)(1 - P_1)}$

$\boxed{c}$ $$P(\bar{W} | \bar{D}_2, \bar{D}_1) = \frac{P(\bar{D}_2, \bar{D}_1 | \bar{W}) P(\bar{W})}{P(\bar{D}_2, \bar{D}_1 | \bar{W}) P(\bar{W}) + P(\bar{D}_2, \bar{D}_1 | W) P(W)}$$

$$= \frac{(1-2P_2)(1-P_1)\frac{1}{2}}{(1-2P_2)(1-P_1)\frac{1}{2} + (1-P_2)(1-P_1)\frac{1}{2}}$$

$$= \frac{1-2P_2}{1-2P_2+1-P_2}$$

$$= \boxed{\frac{1-2P_2}{2-3P_2}}$$

2 a Sampling with replacement and order does not matter:

$$\binom{n+r-1}{r} \quad \text{with } r=n$$

$$= \binom{n+n-1}{n} = \boxed{\binom{2n-1}{n}}$$

b $\boxed{\dfrac{n}{n^n}}$ = $\dfrac{\text{\# of ways to choose the same number}}{\text{total \# of ways to draw } n \text{ numbers}}$

c $\boxed{\dfrac{n!}{n^n}}$ = $\dfrac{\text{\# of ways to draw } n \text{ unique numbers}}{\text{\# of ways to draw } n \text{ numbers}}$

3

$$f(x) = 2(1+x)^{-3} \quad \text{for } x > 0$$

a

$$F_X(x) = \int_0^x 2(1+t)^{-3}\,dt = \frac{2(1+t)^{-2}}{-2}\Bigg]_0^x$$

$$= -(1+x)^{-2} + (1+0)^{-2}$$

$$= \boxed{1-(1+x)^{-2} \quad \text{for } x>0, \text{ otherwise } 0}$$

b

$$Y = 2X = g(X) \qquad \text{1-1 function}$$

$$g^{-1}(y) = \frac{y}{2}$$

$$\frac{d}{dy}g^{-1}(y) = \frac{1}{2}$$

$$f_Y(y) = f_X(g^{-1}(y))\left|\frac{d}{dy}g^{-1}(y)\right|$$

$$= 2\left(1+\frac{y}{2}\right)^{-3}\frac{1}{2}$$

$$= \boxed{\left(1+\frac{y}{2}\right)^{-3} \quad \text{for } y>0}$$

c

$$Z = 1-(1+X)^{-2} = F_X(X)$$

$\Rightarrow Z \sim Unif(0,1)$ by probability integral transformation

$$\Rightarrow \boxed{f_Z(z) = 1 \quad \text{for } 0 < z < 1}$$

$\boxed{4}$ $\boxed{a}$ $M_x(t) = E(e^{tx}) = \sum_{x=0}^{\infty} e^{tx}\left(\frac{1}{2}\right)^{x+1}$

$$= \frac{1}{2}\sum_{x=0}^{\infty}\left(\frac{1}{2}e^t\right)^x$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{2}e^t}\right]$$

because $t < \log(2)$
by geometric series

$$= \frac{1}{2}\,\frac{2}{2-e^t}$$

$$= (2-e^t)^{-1}$$

$\boxed{b}$ $E(x) = \frac{d}{dt}M_x(t)\Big|_{t=0} = -1(2-e^t)(-e^t)\Big|_{t=0}$

$$= -1(2-1)(-1)$$

$$= \boxed{1}$$

$\boxed{c}$ $\text{median}(x) = F_x^{-1}\left(\frac{1}{2}\right) = \inf\{x : F_x(x) \geq \frac{1}{2}\} = \boxed{0}$

x	f(x)	F(x)
→ 0	1/2	1/2
1	1/4	3/4
⋮	⋮	⋮