**Bios 312: Modern Regression Analysis**

Spring, 2013

**Homework #5**

March 14, 2013

**Written problems due at the beginning of class, Thursday, March 21, 2013.**

Regression model theory

Let *Y*i ~ N(**0 + **1\**X*i, 2) as in simple linear regression.

1. Find the maximum likelihood estimates of **0, **1, and **2. Show your work.

2. Compare the MLEs obtained in (1) to the estimate of obtained for minimizing the residual sums of squares. You do not need to derive the least squares estimates (they are given in the notes).

3. Show that (***X***T***X***)-1***X***T***Y =*** $\hat{B}$where

$$X=\left[\begin{matrix}1&x\_{1}\\1&x\_{2}\\\vdots &\vdots \\1&x\_{n}\end{matrix}\right] Y=\left[\begin{matrix}y\_{1}\\y\_{2}\\\vdots \\y\_{n}\end{matrix}\right] \hat{B} =\left[\begin{matrix}\hat{β}\_{0}\\\hat{β}\_{1}\end{matrix}\right]$$

 and $\hat{β}\_{0}$ and $\hat{β}\_{1}$ are the MLEs found in (1). (Refer back to matrix algebra notes if needed)

4. Let *Y*avg be the mean of new observations taken at fixed, known covariate *X*avg in a simple linear regression model where $Y\_{avg}=\hat{β}\_{0}+\hat{β}\_{1}X\_{avg}$. Find the *Var[Yavg]*. At what value of *X*avg will *Var[Yavg]* be smallest?

5. Let *Y*new be the predicted value of a new *Y* taken at fixed, known covariate *X*new in a simple linear regression model where $Y\_{new}=\hat{β}\_{0}+\hat{β}\_{1}X\_{new}+ϵ$. Find the *Var[Ynew]*. At what value of *X*new will *Var[Ynew]* be smallest?

6. Use the *Var[Yavg]* to construct a (1- confidence interval for the expected value of *Y* taken at *Xavg.* Use the *Var[Ynew]* to construct a (1- confidence interval for the predicted value of a new *Y* at *Xnew.* Which of these two confidence intervals will always be wider? Why?