## Applied Biostatistics II Final Examination Key

Name: $\qquad$ Mailbox: $\qquad$
Instructions: Please provide concise answers to all questions. Rambling answers touching on topics not directly relevant to the question will tend to count against you. Nearly telegraphic writing style is permissible.

The examination is closed book and closed notes. If you come to a problem that you believe cannot be answered without making additional assumptions, clearly state the reasonable assumptions that you make, and proceed.

Problems 1 through 3 pertain to the analyses of data from a hypothetical study investigating the association between sex, age, and race on serum cholesterol. Appendix 1 contains results of these analyses.

1. (10 points) Models A, B, and C each model the effects of age, race, sex, and a race-sex interaction on serum cholesterol. Which of these three models is the better one to use for such purposes? Justify your answer.

Ans: Either of models B and C are appropriate (they are in fact different parameterizations of the same model). Model A is inappropriate because it models a nominal (unordered categorical) variable continuously.
2. Using the model you identified in problem 1, answer the following questions
a. (5 points) What is your best estimate of the expected cholesterol in a black female of age 60 ?

Ans: (In all parts of problem 2, I gave full credit if a student gave the estimate appropriate to the model chosen in problem 1—even if the student chose the inappropriate model A.)

A: This is answered by noting that for such a person Age $=60$, Female $=1$, Race $=2$, Race. Female $=2$ : $116.5246+60 *(1.3119)+1 *(-1.6327)+2 *(-0.8262)+2 *(-3.2766)=185.4003$

B: This is answered by noting that for such a person Age=60, Female=1, Black=1, Asian=0, Black.Female=1, Asian.Female= 0:
$117.731+60 *(1.264)+1 *(-6.696)+1 *(1.571)+0 *(-1.563)+1 *(2.812)+0 *(-6.568)=191.258$
C: This is answered by noting that for such a person Age $=60$, Female $=1$, White $=0$, Asian $=0$, White. Female $=0$, Asian.Female $=0$ :
$119.302+60 *(1.264)+1 *(-3.885)+0 *(-1.571)+0 *(-3.134)+0 *(2.812)+0 *(-9.379)=191.258$
b. (5 points) What is your best estimate of the expected cholesterol in a black male of age 50 ?

Ans: (In all parts of problem 2, I gave full credit if a student gave the estimate appropriate to the model chosen in problem 1—even if the student chose the inappropriate model A.)

A: This is answered by noting that for such a person Age $=50$, Female $=0$, Race $=2$, Race. Female $=0$ : $116.5246+50 *(1.3119)+0 *(-1.6327)+2 *(-0.8262)+0 *(-3.2766)=180.4672$

B: This is answered by noting that for such a person Age $=50$, Female $=0$, Black $=1$, Asian $=0$, Black. Female $=0$, Asian.Female= 0:
$117.731+50 *(1.264)+0 *(-6.696)+1 *(1.571)+0 *(-1.563)+0 *(2.812)+0 *(-6.568)=182.502$

C: This is answered by noting that for such a person Age $=50$, Female $=0$, White $=0$, Asian $=0$, White. Female $=0$, Asian. Female $=0$ :
$119.302+50 *(1.264)+0 *(-3.885)+0 *(-1.571)+0 *(-3.134)+0 *(2.812)+0 *(-9.379)=182.502$
c. ( 5 points) What is your best estimate of the expected cholesterol in a black male of age 51 ?

Ans: (In all parts of problem 2, I gave full credit if a student gave the estimate appropriate to the model chosen in problem 1—even if the student chose the inappropriate model A.)

A: This is answered by noting that for such a person Age=51, Female $=0$, Race $=2$, Race. Female $=0$ :
$116.5246+51 *(1.3119)+0 *(-1.6327)+2 *(-0.8262)+0 *(-3.2766)=181.779$
(This could also have been answered by adding the slope for Age to the answer for part b:
$180.4672+1.3119=181.779)$

B: This is answered by noting that for such a person $\operatorname{Age}=51$, Female $=0$, Black $=1$, Asian= 0, Black.Female $=0$, Asian.Female $=0$ :
$117.731+51 *(1.264)+0 *(-6.696)+1 *(1.571)+0 *(-1.563)+0 *(2.812)+0 *(-6.568)=183.766$
(This could also have been answered by adding the slope for Age to the answer for part b:
$182.502+1.264=1813.766$ )
C: This is answered by noting that for such a person Age=51, Female $=0$, White $=0$, Asian $=0$, White. Female $=0$, Asian.Female $=0$ :
$119.302+51 *(1.264)+0 *(-3.885)+0 *(-1.571)+0 *(-3.134)+0 *(2.812)+0 *(-9.379)=183.766$
(This could also have been answered by adding the slope for Age to the answer for part b:
$182.502+1.264=1813.766$ )
d. (5 points) What is your best estimate of the expected difference in cholesterol when comparing a white female of age 66 to a white female of age 65 ?

Ans: This is just the slope for age (though you could have figured out the two predicted values and subtracted them):

ㅁ: $\mathbf{1 . 3 1 1 9}$
B: 1.264
C: 1.264
e. (5 points) What is your best estimate of the expected difference in cholesterol when comparing an Asian male of age 60 to an Asian male of age 59?

Ans: Again, this is just the slope for age (though you could have figured out the two predicted values and subtracted them):

ㅁ: $\mathbf{1 . 3 1 1 9}$
B: 1.264
C: 1.264
f. (5 points) What is your best estimate of the expected difference in cholesterol when comparing a white male of age 66 to a white female of age 66 ?

Ans: This is asking about the "sex effect" in whites, and because there is a race-sex interaction, deriving the estimates from the slopes can is a little involved, though again you could have figured out the two predicted values and subtracted them:

A: The two individuals differ by 1 unit in Female and 1 unit in Race.Female, and agree on all other modeled coveariates: $-1.6327-3.2766=-4.9093$.

B: The two individuals differ by 1 unit in Female, and agree on all other modeled covariates: -6.696.

C: The two individuals differ by 1 unit in Female and 1 unit in White. Female, and agree on all other modeled covariates: -3.885-2.812 = -6.697.
g. (5 points) What is your best estimate of the expected difference in cholesterol when comparing a black male of age 66 to a white male of age 66 ?

Ans: This is asking about a "race effect" (whites vs. blacks) in males, and because there is a race-sex interaction, deriving the estimates from the slopes can is a little involved, though again you could have figured out the two predicted values and subtracted them:

A: The two individuals differ by 1 unit in Race, and agree on all other modeled coveariates: $\mathbf{- 0 . 8 2 6 2}$.
B: The two individuals differ by 1 unit in Black, and agree on all other modeled covariates: $\mathbf{1 . 5 7 1}$.

h. (5 points) What is your best estimate of the expected difference in cholesterol when comparing a black female of age 66 to a white female of age 66 ?

Ans: This is asking about a "race effect" (whites vs. blacks) in females, and because there is a race-sex interaction, deriving the estimates from the slopes can is a little involved, though again you could have figured out the two predicted values and subtracted them:

A: The two individuals differ by 1 unit in Race and 1 unit in Race.Female, and agree on all other modeled coveariates: $\mathbf{- 0 . 8 2 6 2}+(-3.2766)=-4.1028$.

B: The two individuals differ by 1 unit in Black and by 1 unit in Black.Female, and agree on all other modeled covariates: $1.571+2.812=4.383$.

C: The two individuals differ by -1 unit in White and by $\mathbf{- 1}$ unit in White. Female, and agree on all other modeled covariates: - $(\mathbf{- 1 . 5 7 1})-(-2.812)=4.383$.
i. (5 points) What is your best estimate of the expected difference in cholesterol when comparing a black male of age 66 to a black female of age 66 ?

Ans: This is asking about the "sex effect" in blacks, and because there is a race-sex interaction, deriving the estimates from the slopes can is a little involved, though again you could have figured out the two predicted values and subtracted them:

A: The two individuals differ by 1 unit in Female and 2 units in Race.Female, and agree on all other modeled coveariates: $-1.6327+2 *(-3.2766)=-8.1859$.

B: The two individuals differ by 1 unit in Female and 1 unit in Black.Female, and agree on all other modeled covariates: $\mathbf{- 6 . 6 9 6}+2.812=-3.884$.
$\underline{\text { C }}$ : The two individuals differ by 1 unit in Female, and agree on all other modeled covariates: $\mathbf{- 3 . 8 8 5}$.
3. Using the model you identified in problem 1, answer the following questions.
a. (5 points) Is there statistical evidence that the difference between serum cholesterol in males and females varies by race? Justify your answer, including the $P$ value you used.

Ans: This asks if the association between cholesterol and sex is modified by race. Thus we look at a test for the race-sex interaction:

A: As there is only one covariate modeling the race-sex interaction, we look at the $P$ value for that slope parameter: $P=0.0956$. Thus we do not have enough evidence to suggest a difference among the races with respect to the association between cholesterol and sex.

B: There are two covariates modeling the race-sex interaction: Black.Female and Asian.Female. Thus we have to look at the $P$ value from the test that these two slope parameters are simultaneously zero: $P=0.0398$. Because this $P$ value is less than 0.05 , we reject the null hypothesis of no difference among the races in the association between cholesterol and sex. (Note that the $P$ value for the race-sex interaction is statistically significant, even though neither of the individual $P$ values are significant. This is because the individual $P$ values are comparing blacks to whites and Asians to whites. It turns out that the biggest difference in sex effects is between blacks and Asians-see model C.)

C: There are two covariates modeling the race-sex interaction: White.Female and Asian.Female. Thus we have to look at the $P$ value from the test that these two slope parameters are simultaneously zero: $P=0.0398$. Because this $P$ value is less than 0.05 , we reject the null hypothesis of no difference among the races in the association between cholesterol and sex. (Note that if we were to look at the individual $P$ values for the two covariates modeling the race-sex interaction, the lowest $P$ value is 0.0141 - much lower than the true value of 0.0398 . This is due to the multiple comparisons inherent in looking at pairwise comparisons of the races. With three races, there are three ways to compare them in groups of 2 . If we take the lowest $P$ value from such comparisons and judge that $P$ value against 0.05 , our type $I$ error rate will exceed 0.05 . While we could have used a Bonferroni correction by multiplying the lowest $P$ value by 3 , that would have been too conservative: $3 * 0.0141=0.0423$. The best thing to do is use the $P$ value simultaneously testing that all covariates modeling a race-sex interaction are zero: $P=0.0398$.)
b. (5 points) Is there statistical evidence that the difference between serum cholesterol among whites, blacks, and Asians varies by sex? Justify your answer, including the $P$ value you used.

Ans: This asks if the association between cholesterol and race is modified by sex. Thus we look at a test for the race-sex interaction (note that if sex modifies the association between cholesterol and race, then race also modifies the association between cholesterol and sex):

A: As there is only one covariate modeling the race-sex interaction, we look at the $P$ value for that slope parameter: $P=0.0956$. Thus we do not have enough evidence to suggest a difference between the sexes with respect to the association between cholesterol and race.

B: There are two covariates modeling the race-sex interaction: Black.Female and Asian.Female. Thus we have to look at the $P$ value from the test that these two slope parameters are simultaneously zero: $P=0.0398$. Because this $P$ value is less than 0.05 , we reject the null hypothesis of no difference between the sexes in the association between cholesterol and race. (Note that the $P$ value for the race-sex interaction is statistically significant, even though neither of the individual $P$ values are significant. This is because the individual $P$ values are comparing blacks to whites and Asians to whites. It turns out that the biggest difference in sex effects is between blacks and Asians-see model C.)

C: There are two covariates modeling the race-sex interaction: White.Female and Asian.Female. Thus we have to look at the $P$ value from the test that these two slope parameters are simultaneously zero: $P=0.0398$. Because this $P$ value is less than 0.05 , we reject the null hypothesis of no difference between the sexes in the association between cholesterol and race. (Note that if we were to look at the individual $P$ values for the two covariates modeling the race-sex interaction, the lowest $P$ value is 0.0141 - much lower than the true value of 0.0398 . This is due to the multiple comparisons inherent in looking at pairwise comparisons of the races. With three races, there are three ways to compare them in groups of 2 . If we take the lowest $P$ value from such comparisons and judge that $P$ value against 0.05 , our type $I$ error rate will exceed 0.05 . While we could have used a Bonferroni correction by multiplying the lowest $P$ value by 3 , that would have been too conservative: 3 * $0.0141=\mathbf{0 . 0 4 2 3}$. The best thing to do is use the $\mathbf{P}$ value simultaneously testing that all covariates modeling a race-sex interaction are zero: $P=0.0398$.)
c. ( 10 points) Is there a statistically significant difference between the answers to (f) and (i) in problem 2 above? Justify your answer, including the $P$ value you used and how you obtained it.

Ans: This asks if the association between cholesterol and sex in whites is the same as the association between cholesterol and sex in blacks. Thus this is a more restricted question than that in part a when we use dummy variables, but not when the race-sex interaction is modeled by a single continuous variable as in model A. In the presence of the multiple covariates used to model the race-sex interaction with dummy variables, it is common (but not universal) to adjust for multiple comparisons.
$\underline{\text { A }}$ : As there is only one covariate modeling the race-sex interaction, the existence of any such interaction would argue that there is a difference between the sex effect in whites and blacks. So the $P$ value for this question is just the $P$ value for that slope parameter: $P=0.0956$. Thus we do not have enough evidence to suggest a difference between whites and blacks with respect to the association between cholesterol and sex.

B: There are two covariates modeling the race-sex interaction: Black.Female and Asian.Female. Our current question relates only to the value of Black.Female in this parameterization, and thus we base our decision on that $P$ value. We do have to consider whether we should adjust for multiple comparisons: There are three pairwise comparisons of the races that could be made. We are looking at just one of them. A Bonferroni correction would thus multiply the $P$ value by 3 to obtain: $P=3 * .4589$, so I would just write $P>0.5$. Of course, if this comparison between whites and blacks was the only one that would ever have been of scientific interest, adjustment for multiple comparisons would not be necessary and $P=0.4589$. In either case, we do not have enough evidence to suggest a difference between whites and blacks with respect to the association between cholesterol and sex.
$\underline{C}$ : There are two covariates modeling the race-sex interaction: White. Female and Asian.Female. Our current question relates only to the value of White. Female in this parameterization, and thus we base our decision on that $P$ value. We do have to consider whether we should adjust for multiple comparisons: There are three pairwise comparisons of the races that could be made. We are looking at just one of them. A Bonferroni correction would thus multiply the $P$ value by 3 to obtain: $P=3 * .4589$, so I would just write $P>0.5$. Of course, if this comparison between whites and blacks was the only one that would ever have been of scientific interest, adjustment for multiple comparisons would not be necessary and $P=0.4589$. In either case, we do not have enough evidence to suggest a difference between whites and blacks with respect to the association between cholesterol and sex.
4. Appendix 2 contains results from analyses of a hypothetical randomized clinical trial comparing three doses of a new drug with respect to systolic blood pressure. Use the information contained in those results to answer the following questions. You may assume that the necessary assumptions for linear regression are valid for any assumption for which there is no direct information contained in the output. Where appropriate, please identify the regression model you used to answer each question.
a. (5 points) Is there evidence of a statistically significant imbalance in the randomization groups with respect to sex? Justify your answer.

Ans: Using the chi squared analysis, $P=0.1353$, thus we cannot reject the null hypothesis that in the population (which one?) there is no association between sex and dose.
b. (5 points) Is there evidence of confounding by sex on the effect of dose on systolic blood pressure? Justify your answer.

Ans: There is a slight trend toward a higher proportion of males among the high dose groups. There is also a trend toward men having higher blood pressure on average (see model G, which suggests that men on average have a systolic blood pressure 11.29 mm Hg higher than females across all dose groups). Thus sex is associated with our predictor of interest (dose) and our response (systolic blood pressure), and may well confound our detection of an association between average blood pressure and dose. This is further demonstrated by the fact that when we model dose continuously and do not adjust for sex, each 1 unit increase in dose is estimated to cause a 0.2739 mm Hg drop in average blood pressure. However, when we do adjust for sex, each 1 unit increase in dose is estimated to cause a 0.3975 mm Hg drop in average blood pressure. Because we are comparing means, we can use the discrepancy between an unadjusted and an adjusted analysis to detect confounding. This $50 \%$ increase in the estimated effect of the drug would seem to be of scientific importance across a $20 \mathrm{mg} /$ day difference in doses. (Note that the lack of statistical significance in part a is completely irrelevant to this question. Statistical significance plays no role in deciding about confounding. There need not be statistical significance between the confounder and the predictor of interest, because we are interested about associations that exist in the sample; statistical significance tells us about associations that exist in the population. And there need not be statistical significance between the confounder and the response, because we may lack power to detect the association, especially if evaluated in the presence of our predictor of interest: Associations between the confounder and our predictor of interest lessen our ability to detect either association with the response. I do note that usually there will be statistically significant associations between at least one of the comparisons, i.e., either the confounder and the predictor of interest or the confounder and the response. But there are no guarantees-we too often do underpowered studies.)
c. (5 points) Suppose you decided to model dose as dummy variables. Is there evidence of an effect of the drug on blood pressure? Justify your answer, including the model used to address the question and the $P$ value you used to make a decision. Provide a brief interpretation for the parameters in that model.

Ans: You had two basic choices here: Not adjusted for sex, or adjusted for sex. The decision should be made prior to looking at the data. As a general rule, good scientific practice would dictate prespecifying any variables that you would adjust for. Purists would say that you only adjust for variables used in the randomization, thus the unadjusted analyses would be most appropriate. However, knowing that sex might be a strong predictor of blood pressure, but also knowing that stratified randomization is logistically more of a pain than unstratified randomization, it might have been the case that no stratification was done. In such a case, it would not be unusual to prespecify that analyses would still be done after adjustment for sex. In any case, $I$ allowed either approach. I note that models $E$ and $F$ are just different parameterizations of the same unadjusted model, and that models $I$, $J$, and $K$ are also different parameterizations of the same adjusted model. I personally prefer parameterizations that use the dose $\mathbf{0}$ group as the reference group, so I will use those two as examples.
$\underline{E}$ : The test that neither the dose 10 group nor the dose $\mathbf{2 0}$ group differed on average from the dose $\mathbf{0}$ group suggests that we cannot reject the null hypothesis of no effect of drug on average systolic blood pressure ( $\mathrm{P}=$ 0.1597). In this model, the intercept estimates that the average $S B P$ in the placebo group is 141.3 mm Hg . The slope estimates suggest that the average SBP in the dose $\mathbf{1 0}$ group is $0.0502 \mathbf{~ m m ~ H g}$ higher than that in the placebo group, while the average SBP in the dose 20 group is $5.4788 \mathbf{~ m m ~ H g}$ less than that in the placebo group.

I: After adjusting for sex, the test that neither the dose 10 group nor the dose $\mathbf{2 0}$ group differed on average from the dose 0 group suggests that we can reject the null hypothesis of no effect of drug on average systolic blood pressure $(P=0.0243)$. We thus would conclude with high confidence that the treatment does have an effect on systolic blood pressure. In this model, the intercept estimates that the average SBP for females in
the placebo group is $\mathbf{1 3 6 . 4} \mathbf{~ m m ~ H g}$. The slope estimates suggest that the average SBP for males is $\mathbf{1 2 . 3 5} \mathbf{~ m m}$ $\mathbf{H g}$ higher than that for females in the same dose group, that the average SBP in the dose $\mathbf{1 0}$ group is $\mathbf{1 . 1 8 5}$ mm Hg less than that for patients of the same sex in the placebo group, and that the average SBP in the dose $\mathbf{2 0}$ group is $\mathbf{7 . 9 5} \mathbf{~ m m ~ H g}$ less than that for patients of the same sex in the placebo group.
d. (5 points) Using the analysis you used in part (c), for what dose groups is there a statistically significant difference in average blood pressures? Justify your answer, including P values.

Ans: We definitely have a multiple comparison issue here. We have three ways to compare two dose groups, thus giving ourselves three chances to declare difference among dose groups. In this setting, standard statistical practice is to adjust for multiple comparisons, which I will do by the Bonferroni comparison, i.e., by multiplying each individual $P$ value by 3 . Note that $I$ can use the equivalent models to get all three comparisons.
$E$ and $F$ : Bonferroni corrected unadjusted $P$ values are: 1 ) for dose 10 versus placebo we consider $\mathbf{3 *} .9879$ to suggest $P>0.50$, 2) for dose 20 versus placebo we consider $3 * .0988$ to suggest $P=0.2964$, and 3 ) for dose 20 versus dose 10 we consider $3 * .0957$ to suggest $P=0.2871$. In no case was there a statistically significant difference between the groups.

I and J: Bonferroni corrected sex adjusted $P$ values are: 1) for dose 10 versus placebo we consider 3 *. 7009 to suggest $P>0.50,2)$ for dose 20 versus placebo we consider $3 * .0116$ to suggest $P=0.0348$, and 3 ) for dose 20 versus dose 10 we consider $3 * .0297$ to suggest $P=0.0891$. Thus we are only highly confident that a difference exists between the dose 20 and placebo groups. We cannot be sure that dose 10 would work as well as dose 20 , nor can we be sure that the dose 10 group is completely ineffective (equivalent to placebo). (For what it is worth, I simulated this data under a linear continuous model, hence in truth every dose is at least somewhat effective.)
e. (5 points) Suppose you decided to model dose as a continuous variable. Is there evidence of an effect of the drug on blood pressure? Justify your answer, including the model used to address the question and the P value you used to make a decision. Provide a brief interpretation for the parameters in that model.

Ans: Again you had two basic choices here: Not adjusted for sex, or adjusted for sex. And again the decision should be made prior to looking at the data. All the discussion given for part c holds here as well.

D: The test that SBP does not differ across dose groups is based on the slope parameter for dose. Based on $\mathbf{P}=$ 0.0987 , we cannot reject the null hypothesis of no effect of drug on average systolic blood pressure. In this model, the intercept estimates that the average SBP in the placebo group is $\mathbf{1 4 2 . 2} \mathbf{~ m m ~ H g}$. The slope estimate suggests that the average SBP tends to decrease 0.2739 mm Hg for each $\mathbf{1} \mathbf{~ m g} /$ day of dose.

I: After adjusting for sex, we find a statistically significant trend toward lower average SBP with increasing dose $(P=0.0117)$. In this model, the intercept estimates that the average SBP for females in the placebo group is $137.3 \mathbf{~ m m ~ H g}$. The slope estimates suggest that the average SBP for males is $\mathbf{1 2 . 3 5} \mathbf{~ m m ~ H g ~ h i g h e r ~}$ than that for females in the same dose group, and that the average SBP tends to decrease $0.3975 \mathbf{~ m m ~ H g}$ for each $1 \mathbf{~ m g} /$ day of dose relative to patients of the same sex.
f. (5 points) Using the analysis you used in part (e), for what dose groups is there a statistically significant difference in average blood pressures? Justify your answer, including P values.

Ans: If we take the linear model at face value, the finding about the change in average SBP per unit of dose extends to all dose groups. Hence, in the unadjusted analysis, we cannot be confident of a difference between any two dose groups, and in the adjusted analysis we would claim confidence in declaring differences between any two groups of the same sex that differ in dose. (Of course, I would urge you to be extremely circumspect in presuming that a the true relationship is absolutely linear. Most often, however, we are left to decide the best dose with inadequate precision to be sure which is truly optimal.)
g. (10 points) Which of the two analyses considered in parts (c) and (e) would you prefer? Justify your answer, briefly stating the issues that you considered in making your decision.

Ans: In order to demonstrate a treatment effect, I almost always prefer a linear continuous model for dose, unless I have good reason to suspect a U-shaped trend. Adjustment for sex certainly seems appropriate based on the post hoc analysis, but that decision really should be made beforehand. Likely it would have been prespecified in a real trial. Actually, it is likely that randomization would have been stratified by sex.
h. ( 5 points) Is there evidence of confounding by dose on the effect of sex on systolic blood pressure? Justify your answer. Explain why you might get different answers to this question and part $b$.

Ans: There is a slight trend toward a higher proportion of males among the high dose groups. There is also a trend toward higher doses having lower blood pressure on average (see model G). Thus dose is associated with our predictor of interest (sex) and our response (systolic blood pressure), and may well confound our detection of an association between average blood pressure and sex. This is further demonstrated by the fact that when we model dose continuously and do not adjust for dose, males are estimated to average SBP 11.29 $\mathbf{m m ~ H g}$ higher than females, but after adjusting for dose, males are estimated to average SBP $\mathbf{1 2 . 3 5} \mathbf{~ m m ~ H g}$ higher than females in the same dose group. Of course, if dose is not truly associated with average SBP but sex is, then sex can confound our detection of an association between SBP and dose, while dose will not confound the detection of an association between SBP and sex. In the latter case, we would not have confounding because dose is not associated with the response. The key issue is that confounding is not symmetric.
5. Consider the problem of evaluating the prognostic value of the nadir PSA on time to relapse. Recall that variable obstime measured time until relapse or last follow-up, with variable inrem measuring whether the patient was still in remission at last follow-up. Also recall that everyone was followed a minimum of 24 months, thus we could construct a variable relapse 24 that was an indicator of relapse within 24 months. For each of the following regressions, indicate whether the analysis method would be statistically and scientifically valid. When the analysis is valid, identify the measure of association being compared.
a. A linear regression of observation time (response) on nadir PSA (predictor).

Ans: This is inappropriate, because observation time is censored. We can not in general estimate the mean time to progression in the presence of censored data.
b. A linear regression of nadir PSA (response) on an indicator of relapse within 2 years

Ans: This is valid. We are comparing the difference in mean nadir PSA between patients who did and patients who did not relapse.
c. A logistic regression of an an indicator of relapse within 2 years (response) on nadir PSA (predictor)

Ans: This is valid. We are comparing the ratio of the odds of relapse within 2 years across groups defined by nadir PSA levels.
d. A proportional hazards regression model of time to relapse as measured by obstime on nadir PSA (predictor)

Ans: This is valid. We are comparing the ratio of the instantaneous risk of relapse across groups defined by nadir PSA levels. (This would tend to be my top choice to answer this question, as it makes the most efficient use of the data.)

## APPENDIX 1

```
A simulated observational study of the relationship between serum cholesterol and
age, sex and race. The following measurements were made on 100 subjects:
```

```
age age in years
```

age age in years
race coded race: 1= white, 2= black, 3= Asian
race coded race: 1= white, 2= black, 3= Asian
female coded sex: 0= male, 1= female
female coded sex: 0= male, 1= female
chol serum cholesterol (mg/dl)
chol serum cholesterol (mg/dl)
From those variables, the following additional variables were computed

```
```

race.female = race * female

```
race.female = race * female
white = 1 if white (race=1); 0 otherwise
white = 1 if white (race=1); 0 otherwise
black = 1 if black (race=2); 0 otherwise
black = 1 if black (race=2); 0 otherwise
asian = 1 if Asian (race=3); 0 otherwise
asian = 1 if Asian (race=3); 0 otherwise
white.female= white * female
white.female= white * female
black.female= black * female
black.female= black * female
asian.female= asian * female
```

asian.female= asian * female

```
Presented below are:
1) Selected descriptive statistics
2) Results of selected regression analyses
3) For each regression analysis, results of selected tests of multiple parameters

\section*{Descriptive statistics}

Univariate descriptives:
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & msng & n & freq & & mean & std dev & min & 25\%-ile & median & 75\%-ile & maximum \\
\hline age & 0 & 100 & & & 60.53 & 5.92 & 50.67 & 55.17 & 61.05 & 65.42 & 69.78 \\
\hline chol & 0 & 100 & & & 190.91 & 11.35 & 159.60 & 181.89 & 191.29 & 199.27 & 213.13 \\
\hline race & 0 & 100 & & & & & & & & & \\
\hline Asian & & & 34 & 34\% & & & & & & & \\
\hline Black & & & 33 & 33\% & & & & & & & \\
\hline White & & & 33 & 33\% & & & & & & & \\
\hline sex & 0 & 100 & & & & & & & & & \\
\hline Female & & & & 41\% & & & & & & & \\
\hline Male & & & 59 & 59\% & & & & & & & \\
\hline
\end{tabular}

Frequencies by race and sex:
\begin{tabular}{rrrr}
\hline & White & Black & Asian \\
Male & 19 & 20 & 20 \\
Female & 14 & 13 & 14
\end{tabular}

\section*{APPENDIX 1 (cont.)}

\section*{Regression models}

MODEL A: chol on age, female, race and female-race interaction
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Residual S
\[
\mathrm{N}=100,
\] & \[
\begin{aligned}
& \text { d E } \\
& \text { ist }
\end{aligned}
\] & \[
\begin{aligned}
& =7 . \\
& 27.8
\end{aligned}
\] & \[
\begin{array}{ll}
\text {, } & \mathrm{Mu} \\
\text { on } & 4
\end{array}
\] & \[
\begin{array}{r}
\text { ple } \\
95
\end{array}
\] & quare = p-value & \[
0.0000
\] \\
\hline & coef & std.err & t.stat & p.value & 95\% CIlo & 95\% CIhi \\
\hline Intercept & 116.5246 & 8.4132 & 13.8502 & 0.0000 & 99.822 & 133.2269 \\
\hline age & 1.3119 & 0.1354 & 9.6927 & 0.0000 & 1.043 & 1.5806 \\
\hline female & -1.6327 & 4.2231 & -0.3866 & 0.6999 & -10.017 & 6.7513 \\
\hline race & -0.8262 & 1.2600 & -0.6557 & 0.5136 & -3.328 & 1.6752 \\
\hline male & -3.2766 & 9464 & -1.6835 & 0.0956 & -7 & 0. 5874 \\
\hline
\end{tabular}

Selected hypothesis tests on multiple parameters from Model A
```

Test female= 0.0
race.female= 0.0
F-statistic = 14.3000 on 2 and 95 df, p-value = 0.0000
Test race= 0.0
race.female= 0.0
F-statistic = 4.0080 on 2 and 95 df, p-value = 0.0213

```

\section*{APPENDIX 1 (cont.)}
```

MODEL B: chol on age, female, black, asian and female-black,

``` female-asian interactions
```

Residual Standard Error = 7.4510, Multiple R-Square = 0.5949
N = 100, F-statistic = 22.7600 on 6 and 93 df, p-value = 0.0000
coef std.err t.stat p.value 95% CIlo 95% CIhi
Intercept 117.731 7.9783 14.7564 0.0000 101.888 133.5747
age 1.264 0.1303 9.6987 0.0000 1.005 1.5226
female -6.696 2.6245 -2.5515 0.0124 -11.908 -1.4846
black 1.571 2.3908 0.6570 0.5128 -3.177 6.3184
asian -1.563 2.3905 -0.6539 0.5148 -6.310 3.1838
black.female 2.812 3.7803 0.7438 0.4589 -4.695 10.3186
asian.female -6.568 3.6926 -1.7786 0.0786 -13.900 0.7653

```
Selected hypothesis tests on multiple parameters from Model B
Test black= 0.0
    asian= 0.0
    F-statistic \(=0.8739\) on 2 and 93 df, \(p\)-value \(=0.4207\)
Test black.female= 0.0
    asian.female= 0.0
    F-statistic \(=3.3390\) on 2 and 93 df, \(p\)-value \(=0.0398\)
Test female= 0.0
    black.female= 0.0
    asian.female= 0.0
    F-statistic \(=11.4900\) on 3 and 93 df, p-value \(=0.0000\)
Test black= 0.0
    asian= 0.0
    black.female= 0.0
    asian.female= 0.0
    F-statistic \(=5.3750\) on 4 and 93 df, \(p\)-value \(=0.0006\)

\section*{APPENDIX 1 (cont.)}

MODEL C: chol on age, female, white, asian and female-white, female-asian interactions
```

Residual Standard Error = 7.4510, Multiple R-Square = 0.5949
N = 100, F-statistic = 22.7600 on 6 and 93 df, p-value = 0.0000
coef std.err t.stat p.value 95% CIlo 95% CIhi
Intercept 119.302 7.8397 15.2176 0.0000 103.734 134.870
age 1.264 0.1303 9.6987 0.0000 1.005 1.523
female -3.885 2.7242 -1.4260 0.1572 -9.294 1.525
white -1.571 2.3908 -0.6570 0.5128 -6.318 3.177
asian -3.134 2.3706 -1.3221 0.1894 -7.842 1.573
white.female -2.812 3.7803 -0.7438 0.4589 -10.319 4.695
asian.female -9.379 3.7497 -2.5013 0.0141 -16.825 -1.933

```
Selected hypothesis tests on multiple parameters from Model B
Test white= 0.0
    asian= 0.0
    F-statistic \(=0.8739\) on 2 and 93 df, p-value \(=0.4207\)
Test white.female= 0.0
    asian.female= 0.0
    F-statistic \(=3.3390\) on 2 and 93 df, \(p\)-value \(=0.0398\)
Test female= 0.0
    white.female= 0.0
    asian.female= 0.0
    F-statistic \(=11.4900\) on 3 and 93 df, \(p\)-value \(=0.0000\)
Test white= 0.0
    asian= 0.0
    white.female= 0.0
    asian.female= 0.0
    F-statistic \(=5.3750\) on 4 and 93 df, \(p\)-value \(=0.0006\)

\section*{APPENDIX 2}
```

A simulated randomized clinical trial of a new drug to treat blood pressure.
1 5 0 subjects were randomized to one of three doses. The following
measurements were made:
male coded sex: 0= female, 1= male
dose dose administered (mg/day)
sbp systolic blood pressure at end of study (mm Hg)
From those variables, the following variables were computed
dose0 1 if subject in dose 0 group; 0 otherwise
dose10 1 if subject in dose 10 group; 0 otherwise
dose20 1 if subject in dose 20 group; 0 otherwise
Presented below are:
1) Selected descriptive statistics
2) Results of selected regression analyses
3) For each regression analysis, results of selected tests of multiple
parameters

```

\section*{Descriptive statistics}

Univariate descriptives:


\section*{Frequencies by sex and dose}
\begin{tabular}{lrrrl} 
& Dose 0 & Dose \(\mathbf{1 0}\) & Dose \(\mathbf{2 0}\) \\
& & & \\
Female & 30 & 25 & 20 \\
Male & 20 & 25 & 30
\end{tabular}

Chi square test for association between sex and dose: \(X\)-square \(=4, \mathrm{df}=2\), p -value \(=0.1353\)

\section*{APPENDIX 2 (cont.)}

\section*{Regression models}

MODEL D: sbp on dose
Residual Standard Error \(=16.4900\), Multiple R-Square \(=0.01832\)
\(\mathrm{N}=150\), F -statistic \(=2.7610\) on 1 and \(148 \mathrm{df}, \mathrm{p}\)-value \(=0.0987\)
\begin{tabular}{rrrrrrr} 
& coef & std.err & t.stat & p.value & \(95 \%\) CIlo \(95 \%\) CIhi \\
Intercept & 142.2349 & 2.1282 & 66.832 & 0.0000 & 138.0293 & 146.4406 \\
dose & -0.2739 & 0.1649 & -1.662 & 0.0987 & -0.5997 & 0.0518
\end{tabular}

MODEL E: sbp on dose10, dose20
Residual Standard Error = 16.4900, Multiple R-Square = 0.02465
\(\mathrm{N}=150\), F -statistic \(=1.8570\) on 2 and \(147 \mathrm{df}, \mathrm{p}\)-value \(=0.1597\)
\begin{tabular}{rrrrrrr} 
& coef & std.err & t.stat & p.value & \(95 \%\) CIlo & 95\% CIhi \\
Intercept & 141.3051 & 2.332 & 60.6010 & 0.0000 & 136.697 & 145.913 \\
dose10 & 0.0502 & 3.298 & 0.0152 & 0.9879 & -6.467 & 6.567 \\
dose20 & -5.4788 & 3.298 & -1.6615 & 0.0988 & -11.996 & 1.038
\end{tabular}
```

Test dose10= 0.0
dose20= 0.0
F-statistic = 1.8570 on 2 and 147 df, p-value = 0.1597

```

\section*{MODEL F: sbp on dose0, dose10}
```

Residual Standard Error = 16.4900, Multiple R-Square = 0.02465
N = 150, F-statistic = 1.8570 on 2 and 147 df, p-value = 0.1597
coef std.err t.stat p.value 95% CIlo 95% CIhi
Intercept 135.826 2.332 58.251 0.0000 131.2183 140.43
dose0 5.479 3.298 1.662 0.0988 -1.0380 12.00
dose10 5.529 3.298 1.677 0.0957 -0.9878 12.05
Test dose0= 0.0
dose10= 0.0
F-statistic = 1.8570 on 2 and 147 df, p-value = 0.1597

```

\section*{MODEL G: sbp on male}

Residual Standard Error \(=15.6400\), Multiple R-Square \(=0.1167\) \(\mathrm{N}=150\), F -statistic \(=19.5600\) on 1 and \(148 \mathrm{df}, \mathrm{p}\)-value \(=0.0000\) coef std.err t.stat p.value 95\% CIlo 95\% CIhi
Intercept \(133.85 \quad 1.80674 .130 \quad 0 \quad 130.280 \quad 137.42\)
\(\begin{array}{lllllll}\text { male } & 11.29 & 2.554 & 4.423 & 0 & 6.248 & 16.34\end{array}\)

\section*{APPENDIX 2 (cont.)}

MODEL H: sbp on male, dose
Residual Standard Error \(=15.3500\), Multiple R-Square \(=0.1543\)
\(\mathrm{N}=150\), F -statistic \(=13.4100\) on 2 and \(147 \mathrm{df}, \mathrm{p}\)-value \(=0.0000\)
\begin{tabular}{rrrrrrr} 
& coef & std.err & t.stat & p. value & \(95 \%\) CIlo & \(95 \%\) CIhi \\
Intercept & 137.2933 & 2.2275 & 61.635 & 0.0000 & 132.891 & 141.6954 \\
male & 12.3541 & 2.5413 & 4.861 & 0.0000 & 7.332 & 17.3762 \\
dose & -0.3975 & 0.1556 & -2.554 & 0.0117 & -0.705 & -0.0899
\end{tabular}

MODEL I: sbp on male, dose10, dose20
```

Residual Standard Error = 15.3500, Multiple R-Square = 0.1606
N = 150, F-statistic = 9.3120 on 3 and 146 df, p-value = 0.0000
coef std.err t.stat p.value 95% CIlo 95% CIhi
Intercept 136.363 2.397 56.8989 0.0000 131.627 141.100
male 12.354 2.540 4.8631 0.0000 7.333 17.375
dose10 -1.185 3.080 -0.3848 0.7009 -7.272
dose20 -7.950 3.111 -2.5551 0.0116 -14.099 -1.800

```
Test dose10=0.0
    dose20= 0.0
    F-statistic \(=3.8150\) on 2 and 146 df, p-value \(=0.0243\)

\section*{MODEL J: sbp on male, dose0, dose10}
```

Residual Standard Error = 15.3500, Multiple R-Square = 0.1606
N = 150, F-statistic = 9.3120 on 3 and 146 df, p-value = 0.0000
coef std.err t.stat p.value 95% CIlo 95% CIhi
Intercept 128.414 2.652 48.417 0.0000 123.1721 133.66
male 12.354 2.540 4.863 0.0000 7.3334 17.37
dose0 7.950 3.111 2.555 0.0116 1.8005 14.10
dose10 6.764 3.080 2.196 0.0297 0.6771 12.85
Test dose0= 0.0
dose10= 0.0
F-statistic = 3.8150 on 2 and 146 df, p-value = 0.0243

```

\section*{MODEL K: sbp on male, dose, dose20}
```

Residual Standard Error = 15.3500, Multiple R-Square = 0.1606
N = 150, F-statistic = 9.3120 on 3 and 146 df, p-value = 0.0000
coef std.err t.stat p.value 95% CIlo 95% CIhi
Intercept 135.178 2.515 53.7520 0.0000 130.208 140.1484
male 12.354 2.540 4.8631 0.0000 7.333 17.3747
dose0 1.185 3.080 0.3848 0.7009 -4.902 7.2725
dose20 -6.764 3.080 -2.1962 0.0297 -12.852 -0.6771
Test dose0= 0.0
dose20= 0.0
F-statistic = 3.8150 on 2 and 146 df, p-value = 0.0243

```
```

