1 Comparing two proportions example

1.1 Study Description

- Consider patients who will undergo coronary artery bypass graft surgery (CABG)
- Mortality risk associated with open heart surgery
- Study question: Do emergency cases have a surgical mortality that is different from that of non-emergency cases?
- Population probabilities
  - p_1: Probability of death in patients with emergency priority
  - p_2: Probability of death in patients with non-emergency priority
- Statistical hypotheses
  - $H_0 : p_1 = p_2$ (or OR $= 1$)
  - $H_1 : p_1 \neq p_2$ (or OR $\neq 1$)

1.2 Power and Sample Size

- Prior research shows that just over 10% of surgeries end in death
- Researchers want to be able to detect a 3 fold increase in risk
- For every 1 emergency priority, expect to see 10 non-emergency
- $p_1 = 0.3$, $p_2 = 0.1$, $\alpha = 0.05$, and power $= 0.90$
- Calculate sample sizes using the PS software for these values and other combinations of p_1 and p_2

(p_1, p_2)	**(0.3, 0.1)**	(0.4, 0.2)	(0.03, 0.01)	(0.7, 0.9)
n_1	**40**	56	589	40
n_2	**400**	560	5890	400

1.3 Collected Data

In-hospital mortality figures for emergency surgery and other surgery

	Discharge Status	
Surgical Priority	Dead	Alive
Emergency	6	19
Other	11	100

- $\hat{p}_1 = \frac{6}{25} = 0.24$
- $\hat{p}_2 = \frac{11}{111} = 0.10$

1.4 Statistical Test

- Stat program output

```
                Discharge Status
 Priority Dead  Alive oddsratio    lower    upper  p.value
Emergency    6  19    1.000000       NA       NA   NA
Other       11  100   2.870813  0.946971 8.703085  0.05429

$measure
[1] "wald"

$conf.level
[1] 0.95

$pvalue
[1] "chi2"
```

- Interpretation
  - Compare odds of death in the emergency group $\left(\frac{\hat{p}_1}{1-\hat{p}_1}\right)$ to odds of death in non-emergency group $\left(\frac{\hat{p}_2}{1-\hat{p}_2}\right)$
  - Emergency cases are 2.87 times more likely (95% CI: [0.95, 3.36]) than non-emergency cases to die during surgery.

1.4.1 Fisher's Exact Test

Observed marginal totals from emergency surgery dataset

	Dead	Alive	
Emergency	a	b	25
Other	c	d	111
	17	119	136

- With fixed marginal totals, there are 18 possible tables ($a = 0, 1, \ldots 17$)
- Can calculated probability of each of these tables
  - p-value: Probability of observing data as extreme or more extreme than we collected in this experiment
- Exact test: p-value can be calculated "exactly" (not using the Chi-squared distribution to approximate the p-value)
- Stat program output

```
                two-sided
Surgical Priority midp.exact fisher.exact chi.square
        Emergency         NA           NA         NA
        Other     0.07930086    0.0870594 0.05429257
```

- Fisher's test more conservative than Pearson's Chi-square Test (larger p-value)