

## XI. OTHER TOPICS

### Complicated Statistics with Nasty Properties

#### Bootstrap Analysis

- ❖ Treat the sample as if it were the target population
- ❖ Sample repeatedly without replacement to obtain many samples of the same size as the real sample
- ❖ Calculate the test statistic for each sample
- ❖ Examine the variation of the test statistic among bootstrapped samples to assess its dispersion.

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### Multiple imputation of missing values

- ❖ Most statistical packages, including Stata do complete case analyses. That is they discard the data on any patient who is missing any model covariate.
- ❖ Multiple imputation is a method that adjusts for missing data by predicting missing values from non-missing covariates.
- ❖ Lead to unbiased results if the probability of the outcome of interest is not affected by whether a specific covariate is missing.
- ❖ Stata has a very comprehensive package for doing multiple imputation
- ❖ Particularly useful to adjust for missing values in confounding variables.

## 1. Discriminatory Analysis

We often wish to place patients into two or more groups on the basis of a set of explanatory variables with a minimum of misclassification error.

For example, we might wish to classify patients as

- having or not having cancer,
- benefiting or not benefiting from aggressive therapy.

We typically start off with a learning set of patients whose true classification is known. We then use these patients for developing rules to classify other patients. The three most common ways of doing this are as follows.

- **Logistic Regression**

The **linear predictor** from a multiple logistic regression can be used to develop a **classification rule**. Patients whose linear predictor is greater than some value are assigned to one group; all other patients are assigned to the other.

The advantages of this approach are

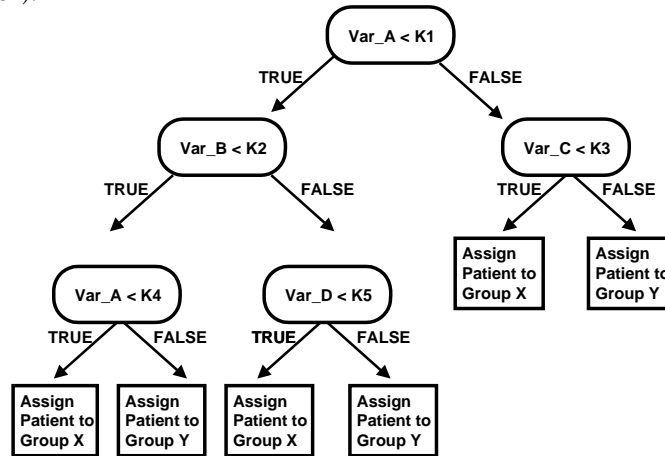
- ⇒ It can lead to a simple rule based on a weighted sum of covariates.
- ⇒ By adjusting the cutoff point we can control the sensitivity and specificity of the rule. It is easy to generate **receiver operating characteristic curves** for this method.
- ⇒ Particularly effective when used with restricted cubic splines

The disadvantage is that the rule may be less than optimal if the model is mis-specified.

- **Classification and Regression Trees**
- **Neural Networks**

## 2. Classification and Regression Trees (CART)

The basic idea here is to derive a tree that consists of a series of binary decisions that lead to patient classification (Breiman et al. 1984).



The CART graphic indicates the degree of increased homogeneity induced by each split. Trees can then be pruned back to produce a classification rule that makes clinical sense and is fairly easy to remember.

The advantages of this method are

- It often does better than logistic regression when the model for the latter is poorly specified.
- It gives a rule that is intelligible to clinicians and can be judged by its clinical criteria.

A disadvantage is that, when applied to continuous covariates it loses information due to the fact that it dichotomizes the selected variable at each split.

### 3. Neural Networks

This method attempts to outperform the logistic regression approach by adopting models that varies from complex to extremely complex (Hinton 1992).

#### Advantages

- Great name.
- Sometimes does better than logistic regression.

#### Disadvantages

- Method is essentially a black box. You need a computer to apply it and it is very difficult to gain intuitive insight into what it is doing.
- Method usually performs only as well as the CART method or logistic regression models with restricted cubic splines.

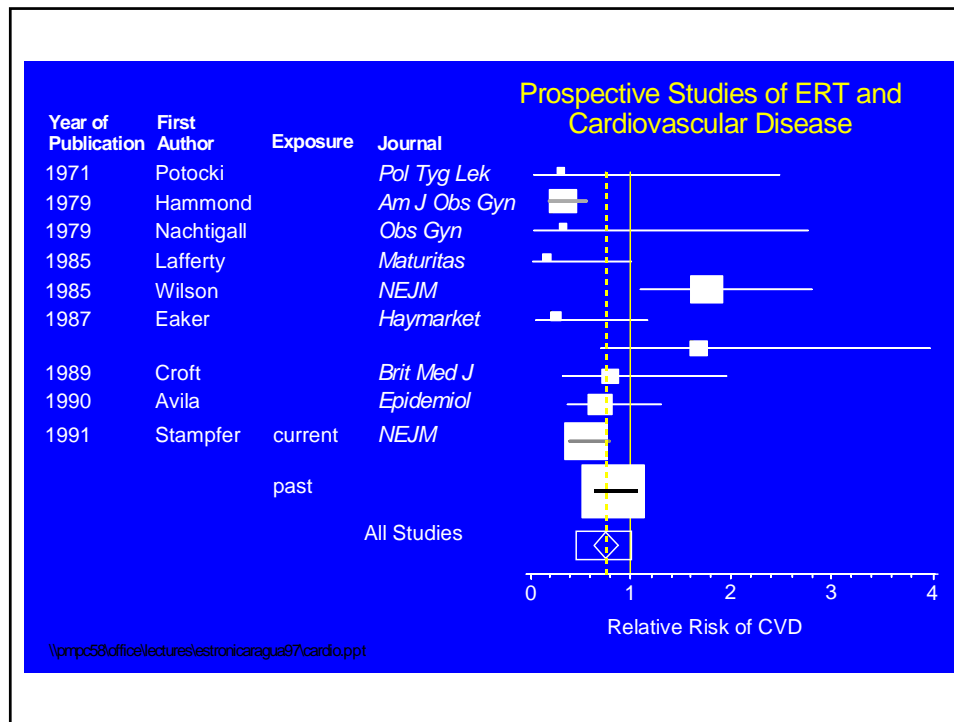
### 4. Meta-Analyses

**One of the strengths of this approach is the meta-analysis graphic.**

This is a rather pretentious term for doing quantitative reviews of the medical literature. The English refer to these techniques as **quantitative overviews**, which is a far more reasonable description. However, in this country we appear to be stuck with the term meta-analysis.

The basic steps in performing a meta-analysis are as follows:

- Systematically identify all publications that may be germane to the topic of interest.
- Review these publications. Eliminate those that are irreverent or misleading using explicitly defined criteria.
- Present the results of the individual studies graphically to show the extent to which they agree or disagree.
- Use clinical judgment and statistical methods to determine whether it is reasonable to combine some or all of the studies into a single analysis. In this case present the relative risk derived from the combined data, together with its 95% confidence interval.



- In these graphs the relative risk from each study is displayed on a single line.
- Each relative risk or odds ratio is plotted as a square.
- The size of this square is proportional to the reciprocal of the variance of the log relative risk (often referred to as the study **information**).
- The 95% confidence interval for each study is depicted as a horizontal line.
- A vertical line depicts a weighted geometric mean of the studies. This mean is weighted by the information content of each study.
- One, or preferably two, 95% confidence intervals are drawn for this combined geometric mean. These confidence intervals are usually drawn as diamonds or squares. They are calculated using either a fixed effects or random effects model.

**a) Fixed effects model for meta-analysis**

This approach assumes that all studies are measuring the same risk in a comparable way, and that the only variation between studies is due to chance.

If this assumption is false it will overestimate the precision of the combined estimate.

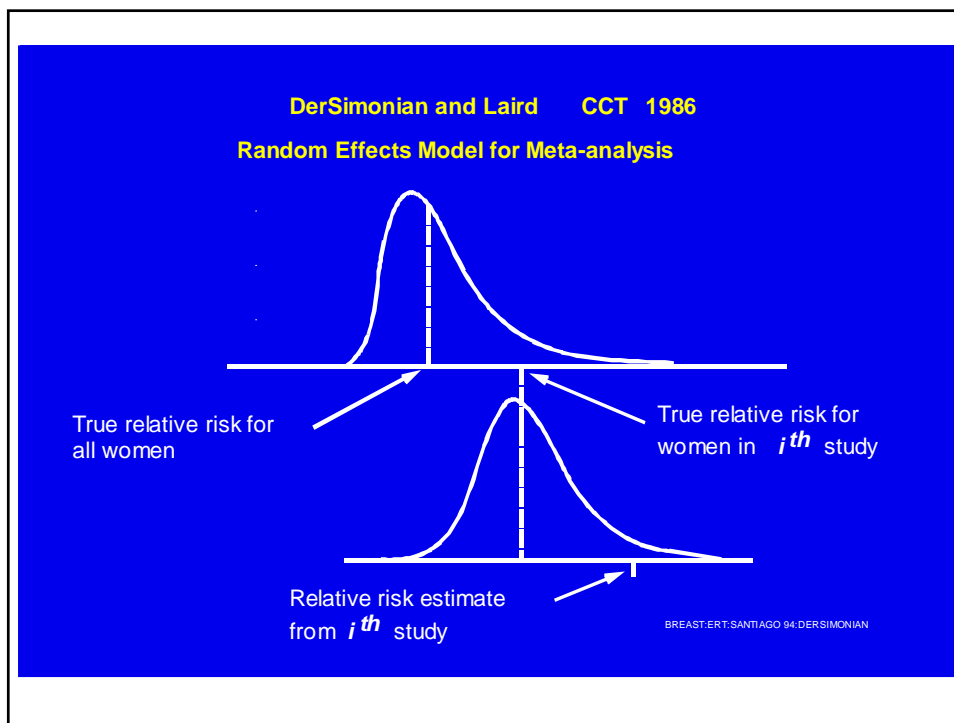
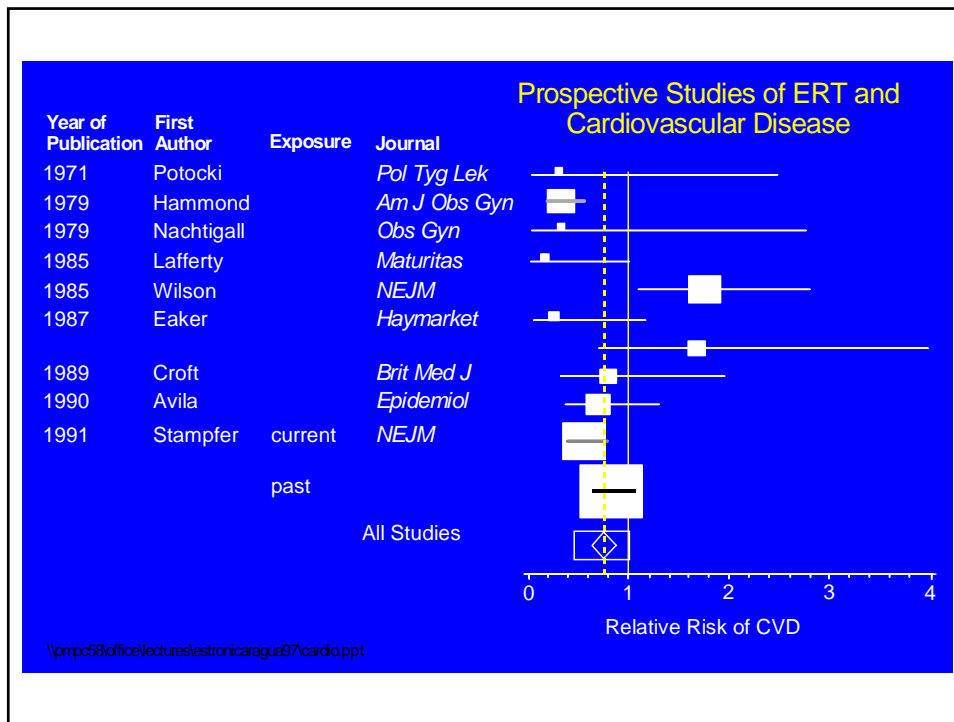
**b) Random effects model for meta-analysis**

This model assumes that each study is estimating a different unknown relative risk that is specific to that study. These risks differ from one study to the next due to differences in study populations, study designs, or biases of one kind or another.

It assumes that these study-specific relative risks follow a log-normal distribution, and that the variation in the estimated relative risks is due both to variation in the study specific risk as well as intra-study variation of study subjects.

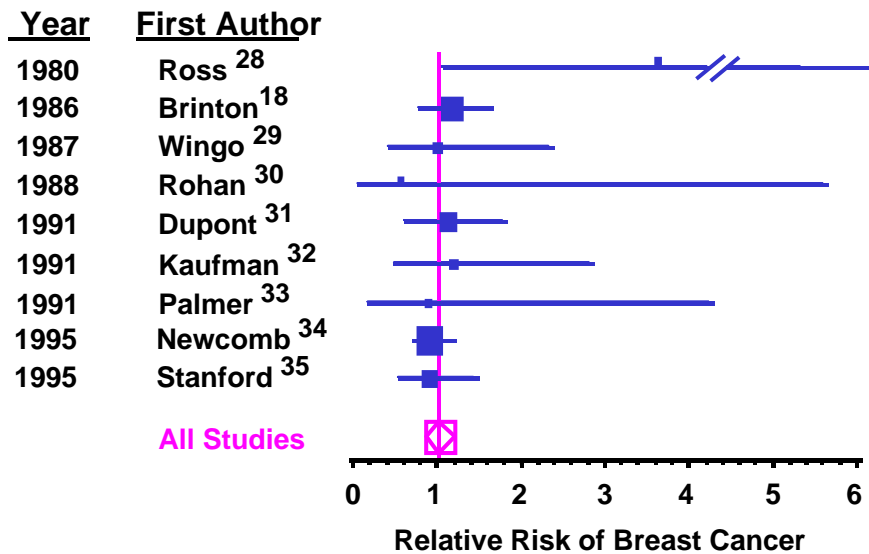
DerSimonian and Laird (1986) devised a way to estimate the confidence interval for the combined relative risk for this model.

It is a good idea to plot both the fixed effects and random effects confidence intervals for the combined relative risk estimate. If these intervals disagree then the inter-study variation is greater than we would expect by chance and the studies are most likely estimating different risks. In this case we need to be very cautious about combining the results of these studies.



On the other hand, if these estimates agree then the studies are mutually consistent and there is no statistical reason not to combine them.

**Women with a History of Benign Breast Disease**  
 Breast cancer risk among ERT users compared to non-users





### 5. Publication bias

One of the ways that meta-analyses can be misleading is through **publication bias**. That is, papers may be more likely to be **published** if they show that a risk factor either **increases or reduces** some risk than if they find a relative risk near one.

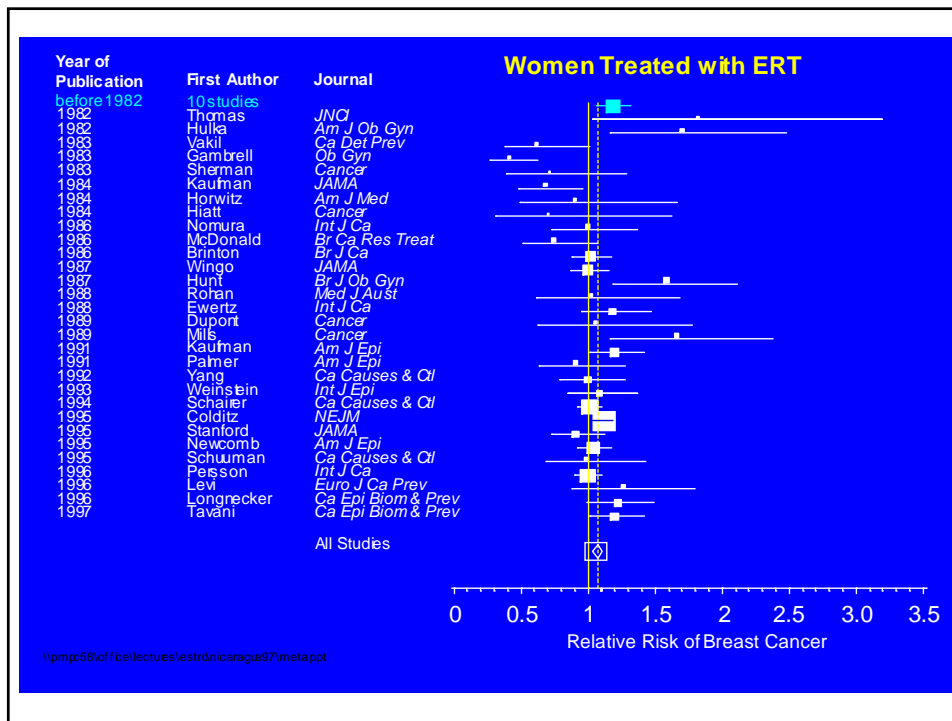
Small studies are more likely to be affected by publication bias than large ones.

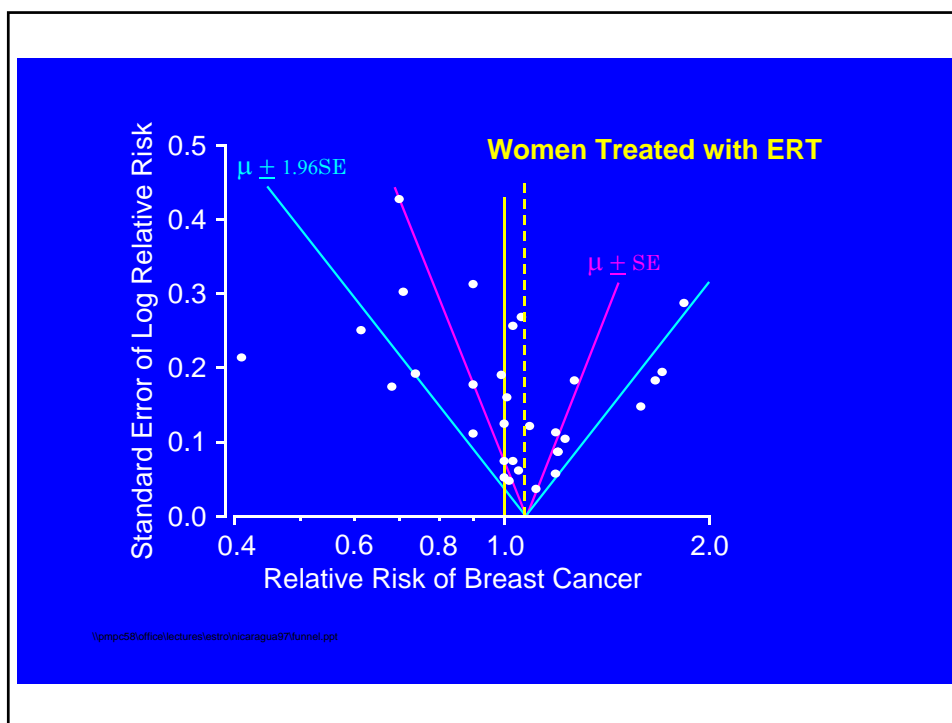
### 6. Funnel graphs

One way to check for publication bias is to plot **funnel graphs** (Light & Pillemer 1984).

In these graphs we plot the **standard error** of the log **relative risk** against **log relative risk**. If this plot has a funnel shape we have evidence of publication bias

When this happens it may make sense to exclude studies with a standard error of the log relative risk that is greater than some value.





### Approaches to Extreme Multiple Comparisons Problems

- ❖ Permutation Tests
- ❖ Cross validation Methods
- ❖ False Discovery Rates
- ❖ Shrinkage Analysis
- ❖ Learning set Test Set Analyses

**This course has been concerned with methods that are appropriate when the number of patients far exceeds the number of model parameters.**

### Diversity Among Statisticians

We all want to

Minimize probabilities of Type I errors

Minimize probabilities of Type II errors

All other things being equal, simple explanations are better than complex ones.

*Science may be described as the art of systematic over-simplification — the art of discerning what we may with advantage omit.*

Karl Popper

Today, reputable statisticians may disagree to some extent about the relative emphasis that should be placed on these three goals

## XII. SUMMARY OF MULTIPLE REGRESSION METHODS

**Table 1.1.** Classification of Response Variables and Regression Models

Nature of Response Variable(s)	Model	Table in Appendix A	Chapters
<b>One response per patient</b>			
Continuous	Linear regression	A.1	2, 3, 10
Dichotomous	Logistic regression	A.2	4, 5
Categorical	Proportional odds and polytomous logistic regression	A.2	5
Survival	Hazard regression	A.3	6, 7
Rates	Poisson regression	A.4	8, 9
<b>Multiple responses per patient</b>			
Continuous	Response feature and generalized estimating equation analysis	A.5	11
Dichotomous	Response feature and generalized estimating equation analysis	A.5	11

**Table A.1.** Models for continuous response variables with one response per patient.

Model Attributes	Method of Analysis	Pages
Normally distributed response variable.		
Single continuous independent variable.		
Linear relationship between response and independent variable.	Simple linear regression.	47 – 99
Non-linear relationship between response and independent variable.	Multiple linear regression using restricted cubic splines.	138 – 159
	Transform response or independent variables and use simple linear regression.	75 – 84
	Convert continuous independent variable to dichotomous variables and use multiple linear regression.	222 – 231, 100 – 163
Single dichotomous independent variable.	Independent <i>t</i> -test.	36 – 41
Single categorical variable.	Convert categorical variable to dichotomous variables and use multiple linear regression.	222 – 231, 100 – 163
	One-way analysis of variance.	439 – 457

**Table A1.** Continued: continuous response, fixed effects

Model Attributes	Method of Analysis	Pages
Normally distributed response variable.		
Multiple independent variables.		
Independent variables have additive effects on response variable.	Multiple linear regression model without interaction terms.	100 – 124
Independent variables have non-additive effects on response variable.	Include interaction terms in multiple linear regression model.	111 – 114
Independent variables are categorical or have non-linear effects on the response variable.	Multiple linear regression: see above for single independent variable.	100 – 163
Two independent categorical variables.	Two-way analysis of variance.	457 – 458
Multiple categorical and continuous independent variables.	Analysis of covariance. This is another name for multiple linear regression.	100 – 163

Table A1. Continued: continuous response, fixed effects

Model Attributes	Method of Analysis	Pages
Skewed response variable.		
Single dichotomous independent variable.	Wilcoxon-Mann-Whitney rank-sum test.	446
Single categorical independent variable.	Kruskal-Wallis test.	445 – 446
Any combination of independent variables.	Apply normalizing transformation to response variable. Then see methods for linear regression noted above.	75 – 84

Table A.2. Models for dichotomous or categorical response variables with one response per patient.

Model Attributes	Method of Analysis	Pages
Dichotomous response variable.		
Single continuous independent variable.		
Linear relationship between log-odds of response and independent variable.	Simple logistic regression.	164 – 206
Non-linear relationship between log-odds of response and independent variable.	Multiple logistic regression using restricted cubic splines.	271 – 285
	Transform independent variable. Then use simple logistic regression.	75 – 84, 164 – 206
	Convert continuous variable to dichotomous variables and use multiple logistic regression.	222 – 230
Single dichotomous independent variable.	2 × 2 contingency table analysis. Calculate crude odds ratio.	193 – 197
	Simple logistic regression.	197 – 203
Single categorical variable.	Convert categorical variable to dichotomous variables and use multiple logistic regression.	222 – 231

Table A2. Continued: dichotomous response, fixed effects

Model Attributes	Method of Analysis	Pages
Dichotomous response variable.		
Multiple independent variables.		
Two dichotomous independent variables with multiplicative effects on the odds ratios.	Mantel-Haenszel odds-ratio and test for multiple $2 \times 2$ tables.	207 – 216
	Multiple logistic regression.	218 – 224
Independent variables have multiplicative effects on the odds-ratios.	Multiple logistic regression model without interaction terms.	216 – 238
Independent variables have non-multiplicative effects on the odds-ratios.	Include interaction terms in multiple logistic regression model.	238 – 244
Independent variables are categorical or have non-linear effects on the log odds.	Multiple logistic regression. See above for single independent variable.	222 – 231, 271 – 285, 75 – 84
Matched cases and controls.	Conditional logistic regression.	264 – 265

Table A2. Continued: categorical response, fixed effects

Model Attributes	Method of Analysis	Pages
Dichotomous response variable.		
Categorical response variable.		
Response categories are ordered and proportional odds assumption is valid.	Proportional odds logistic regression.	285 – 287
Response categories not ordered or proportional odds assumption invalid.	Polytomous logistic regression.	287 – 289
Independent variables have non-multiplicative effects on the odds-ratios, are categorical or have non-linear effects on the log odds.	See above for logistic regression.	238 – 244, 222 – 231, 271 – 285, 75 – 84

**Table A.3.** Models for survival data (follow-up time plus fate at exit observed on each patient).

Model Attributes	Method of Analysis	Pages
Categorical independent variable.	Kaplan-Meier survival curve.	298 – 305
	Log-rank test.	305 – 314
Proportional hazards assumption valid.		
Single continuous independent variable.		
Linear relationship between log-hazard and independent variable.	Simple proportional hazards regression model.	315 – 321
Non-linear relationship between log-hazard and independent variable.	Multiple proportional hazards model using restricted cubic splines.	329 – 332, 341 – 357
	Transform independent variable. Then use simple proportional hazards model.	75 – 84, 315 – 321
	Convert continuous variable to dichotomous variables. Then use multiple proportional hazards regression model.	332 – 333, 341 – 357
Time denotes age rather than time since recruitment.	Proportional hazards regression analysis with ragged entry.	358 – 363

Table A3. Continued: survival data

Model Attributes	Method of Analysis	Pages
Proportional hazards assumption valid.		
Single categorical independent variable.	Convert categorical variable to dichotomous variables and use multiple proportional hazards regression model.	222 – 224, 332 – 333, 341 – 357
	Multiple independent variables.	
	Independent variables have non-multiplicative effects on the hazard ratios.	Include interaction terms in multiple proportional hazards regression.
Independent variables are categorical or have non-linear effects on the log-hazard.	Multiple proportional hazards regression. See above for single independent variable.	324 – 368

Table A3. Continued

Model Attributes	Method of Analysis	Pages
Proportional hazards assumption invalid.	Stratified proportional-hazards regression analysis.	357 – 358
	Hazard regression analysis with time-dependent covariates.	368 – 379
Events are rare and sample size is large.	Poisson regression.	393 – 436
Independent variables have non-multiplicative effects on the hazard ratios.	Include interaction terms in time-dependent hazard regression model.	336 – 337,
		368 – 379
Independent variables have non-linear effects on the log-hazard.	See above for a single continuous independent variable. Use a time-dependent hazard regression model.	329 – 332,
		75 – 84, 368 – 379
Independent variables are categorical.	Convert categorical variables to dichotomous variables in time-dependent model.	332 – 333,
		368 – 379
Time denotes age rather than time since recruitment	Hazards regression analysis with time-dependent covariates and ragged entry.	358 – 363,
		368 – 379

**Table A.4.** Models for response variables that are event rates or the number of events during a specified number of patient-years of follow-up. The event must be rare.

Model Attributes	Method of Analysis	Pages
Single dichotomous independent variable.	Incident rate ratios.	383 – 386
	Simple Poisson regression.	387 – 391
Single categorical independent variable.	Convert categorical variable to dichotomous variables and use multiple Poisson regression.	222 – 224,
		414 – 432
Multiple independent variables.		
Independent variables have multiplicative effects on the event rates.	Multiple Poisson regression models without interaction terms.	411 – 417
		417 – 432
Independent variables have non-multiplicative effects on the event rates.	Multiple Poisson regression models with interaction terms.	417 – 432
		222 – 224,
Independent variables are categorical.	Multiple Poisson regression. See above for single independent variable	414 – 432



**Table A.5.** Models with multiple observations per patient or matched or clustered patients.

Model Attributes	Method of Analysis	Pages
<b>Continuous response measures.</b>		
Dichotomous independent variable.	Paired <i>t</i> -test.	33 – 36
<b>Multiple independent variables.</b>		
	Response feature analysis: consider slopes of individual patient regressions or areas under individual patient curves.	469 – 479
	GEE analysis with identity link function and normal random component.	479 – 491
<b>Dichotomous response measure.</b>		
Multiple independent variables	Response feature analysis: consider within-patient event rate.	470
	GEE analysis with logit link function and binomial random component.	491

Problem	Method
<b>Cross-sectional Study</b>	
Continuous outcome	
Normally distributed	
Linear model ok	Linear regression Fixed-effects analysis of variance
Non-linear model	Linear model of transformed data Linear model with restricted cubic splines
Skewed response data	Linear model of transformed data
Dichotomous outcome	
Rare response	Logistic regression Poisson regression
<b>Longitudinal Data</b>	
	Response feature analysis Repeated measures analysis of variance Generalized estimating equation analysis

Problem	Method
<b>Cohort Study</b>	
Proportional hazards assumption ok	Hazard regression
Rare events	Poisson regression
Ragged entry	Proportional hazard regression with ragged entry times
Expensive data collection	Logistic regression (Nested case-control study)
Complete follow-up with time to failure not important	Logistic regression
Proportional hazards invalid	Stratified hazard regression
Entry uniform or ragged	Time dependent hazard regression Poisson regression
Large study: proportional hazards assumption invalid	Poisson regression
<b>Case-Control Study</b>	
Unstratified or large strata	Unconditional logistic regression
Small strata	Conditional logistic regression

### Additional Reading

A good reference for the response-compression approach to mixed-effects analysis of variance is Matthews et al. (1990).

Classic although rather mathematical references for generalized estimating equations are Liang and Zeger (1986) and Zeger and Liang (1986). Diggle et al. (2002) is an authoritative text on the analysis of longitudinal data.

Armitage and Berry (1994) discuss receiver operating characteristic curves.

Classification and regression trees are discussed by Breiman et al. (1984).

An introduction to neural networks is given by Hinton (1992). A comparison of neural nets with classification and regression trees is given by Reibnegger et al. (1991)

An introduction to meta-analysis is given by Greenland (1987). This paper also describes the fixed effects method of calculating a confidence interval for the combined relative risk estimate. The random effects method is given by DerSimonian and Laird (1986).

Harrell (2001) is an advanced text on modern regression methods.

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For additional references see

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