What do you believe about θ?
P(0.49 < θ < 0.51) = ?

The Beta distribution:
Prior(θ) = p(θ) = p(θ | α, β) = θ ^(α-1) * (1-θ)^(β-1) / B(α, β)

Explore it in R using rbeta(10^6, α, β).
hist(rbeta(10^6, 1, 1), xlim=c(0,1))

Let X = the number of heads and X ~ Binomial(n, θ). Thus,
f(x | θ) = choose(n, x) * θ^x * (1-θ)^(n-x)

Now that you've collected some data, i.e. you have an x value, how does your belief about θ change?

Apply Bayes Theorem to solve for f(θ | x).

f(θ | x) = f(x | θ) * p(θ) / k_1 -- by Bayes theorem
where k_1 = f(x) = $\int_0^1 f(x \mid \theta)\ p(\theta)\ d\theta$
Note for a given value of x, k_1 is just a constant.

f(θ | x) = θ^x * (1-θ)^(n-x) * θ ^(α-1) * (1-θ)^(β-1) / k_2
where k_2 = k_1/choose(n, x). k_2 is still a constant for a given x.

f(θ | x) = θ ^(α-1 + x) * (1-θ)^(β-1 + n-x) / k_2

We know the integral of f(θ | x) for θ from 0 to 1 must equal 1, so we can rewrite the constant as … aha!
f(θ | x) = θ ^(α+x-1) * (1-θ)^(β+n-x-1) / B(α+x, β+n-x)

Try it:
Prior(θ) = p(θ) = p(θ | α, β) = Beta(α, β)

Let α=15 and β=15
hist(rbeta(10^6, 15, 15), xlim=c(0,1))

Now observe some data. What does your belief look like now?
f(θ | x) = θ ^(α+x-1) * (1-θ)^(β+n-x-1) / B(α+x, β+n-x)
= Beta(α+x, β+n-x)

f = rbeta(10^6, 15+? , 15+?)
hist(f, xlim=c(0,1))

The distribution graphically shows your full belief distribution. How could you summarize your updated belief about θ with a point estimate and an interval?

Can you answer the question P(0.49 < θ < 0.51) = ?
mean(0.49 < f & f < 0.51)

Observe some more data. Now what do you believe?

What would it take to dramatically change your belief about θ?