

Exam 1, Biostatistics 341

22 September, 2014

Please show all your work and perform all calculations to whatever degree of exactness you are able. This test is closed book and no calculators are allowed. Please justify your answers. Partial credit will be given.

1. (25) The probability of death for women during a dangerous transplantation surgery is p_1 . Among those women who survive surgery, the probability of rejecting the transplantation and dying within the first two months post-surgery is p_2 .

a. What is the probability that a woman undergoing the surgery will survive 2 months?

b. Men have an identical probability of death during surgery but they are two times more likely to die in the 2 months post-surgery than women. What is the probability that a man undergoing surgery will survive 2 months?

c. Assume men and women are equally likely to get the surgery. Given a person survives 2 months post-surgery, what is the probability that he/she is male?

2. (25) A dataset of size n consists of n unique numbers (i.e., there are no repeats). With the bootstrap, one samples n numbers from the dataset with replacement. The order of the numbers in the bootstrap sample does not matter. For example, suppose the original dataset is $\{1, 2, 3, 4\}$. A bootstrap sample could be $\{1, 1, 3, 4\}$ (same number selected twice because sampling with replacement), and the bootstrap sample $\{1, 2, 3, 4\}$ is equivalent to $\{4, 3, 2, 1\}$.

a. How many possible unique bootstrap samples are there?

b. What is the probability that every number in the bootstrap sample is the same?

c. What is the probability that every number in the bootstrap sample is different?

3. (25) Let X be a random variable with probability density function $f(x) = 2(1+x)^{-3}$ for $x > 0$.

a. What is the cumulative distribution function for X ?

b. Suppose $Y = 2X$. What is the probability density function for Y ?

c. Suppose $Z = 1 - (1 + X)^{-2}$. What is the probability density function for Z ? (Hint: Part (a) might help you.)

4. (25) Let X be a random variable with probability mass function $f(x) = \left(\frac{1}{2}\right)^{x+1}$ for $x = 0, 1, 2, \dots$

a. Show that the moment generating function of X , $M_x(t)$, is $(2 - e^t)^{-1}$. (Assume that $t < \log(2)$.)

b. What is the expectation of X ?

c. What is the median of X ? (Use Casella & Berger's definition for the inverse CDF.)